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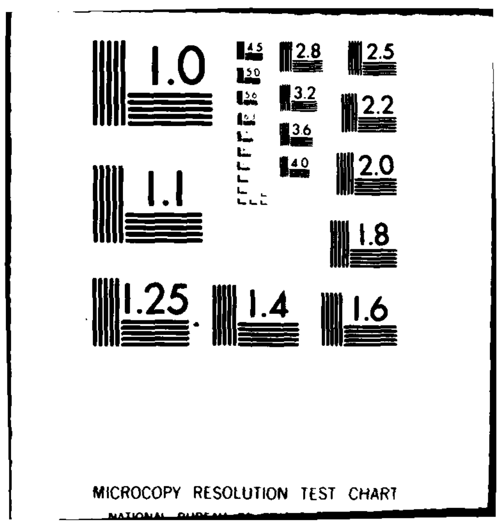
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COMPARISON OF POTENTIAL COEFFICIENT DETERMINATIONS WITH 5° AND 1° ANOMALIES

Richard H. Rapp

The Ohio State University
Research Foundation
Columbus, Ohio 43212

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Potential coefficients can be derived from mean gravity anomalies, satellite data analysis, or a combination of both data types. In using gravity data, a choice of the size of the mean anomaly block to use must be made. Usually this has been 5° equal area blocks as opposed to smaller size blocks such as 1° x 1°. Tests are described in this paper that have been made using 1° x 1° anomalies and 5° anomalies that are consistent with the 1° data. Potential coefficients have been deter-

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mined with just the anomaly data, and the anomaly data in combination with the GEM 9 potential coefficients. In the combination solution the average percentage difference between the solutions using the two anomaly sizes was 29%. The root mean square undulation difference was ±1.1 m and the root mean square anomaly difference was ±3.8 mgals. These differences are caused by the perturbation of the low frequency information by the high frequency information in the mean anomaly blocks. These differences suggest that for the highest accuracy, even if coefficients just to degree 36 are sought, 1° x 1° anomaly blocks should be used.

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We have also examined an approximate technique for the combination solution using 1° x 1° data that requires a significantly less amount of computer time than the rigorous solution. In comparing two 180 x 180 fields from a rigorous and approximate adjustment we found an average percentage difference of 9%; a RMS undulation difference of 0.8 m; and a RMS anomaly difference of 2 mgals.

Foreword

This report was prepared by Richard H. Rapp, Department of Geodetic Science, The Ohio State University, under Air Force Contract No. F19628-79-C-0027, The Ohio State University Research Foundation Project No. 711664. The contract covering this research is administered by the Air Force Geophysics Laboratory, Hanscom Air Force Base, Massachusetts, with Mr. Bela Szabo, Contract Monitor.

The author acknowledges the extensive programming support given to the work by Mr. Kostas Katsambalos, Graduate Research Associate, Department of Geodetic Science.

Introduction

In the past few years our knowledge of the earth's gravitational field has increased significantly. Part of this improvement has come from combining terrestrial surface gravity information with satellite derived data. This merger has enabled a solution where the lower degree (and resonance) terms have dominantly been determined from satellite data while the higher degree terms have dominantly been inferred from the terrestrial data.

In carrying out these combination solutions it has been customary to use the gravity data in the form of averages in 5° equal area blocks. Recent such solutions have been described by, for example, Lerch et al. (1979). In these solutions potential coefficients have been determined to degree 36. But we should ask, why use 5° anomalies in these solutions instead of a smaller size such as 1° blocks. One reason relates to the computational effort required for the processing of such data. There are 1654 5° equal area anomalies but approximately 42000 1° equal area values or 64800 $1^\circ \times 1^\circ$ equiangular blocks on the surface of the earth. Thus a practical limitation may exist which is implemented by solving for coefficients only to a certain maximum degree. This maximum degree has increased in the past few years but with current programs it is usually thought that the highest degree that could be found would be found from the $180^\circ/\theta^\circ$ rule. Thus for 5° blocks the maximum degree to be found was 36. However, Rapp (1977) suggested that the $180^\circ/\theta^\circ$ rule was not strictly valid so that this justification needs to be re-examined.

In our general process we are trying to recover potential coefficients from area mean values of gravity anomalies (ignoring the satellite contributions for a moment). Contributions from all frequencies in the spherical harmonics are made to the area means. Clearly, averaging damps out the high frequencies but it does not eliminate them with the standard averaging operator. This high frequency information within the means can cause perturbations of the low frequency information when solutions are made to extract the coefficients.

By going to a smaller block size we approach the theoretical requirement of having infinitely small blocks. In this case there will be no low frequency perturbations caused by the finite block size. In practice, infinitely small blocks can not be found (on a global basis) so we would expect, as we go to smaller blocks, the perturbation on the low degree coefficients would decrease but not be completely eliminated.

To examine these effects we will carry out several solutions for potential coefficients using surface gravity data alone and then in combination solutions with different block sizes. We will then compare various solutions to see the extent of the differences caused by the use of different block size.

Potential Coefficients from Anomaly Data Alone

In a spherical approximation, gravity anomalies, Δg, are related to fully normalized potential coefficients $(\bar{C}, \bar{S})$ through the following.

$$\left\{\begin{matrix}\bar{C}_{\ell m}\\ \bar{S}_{\ell m}\end{matrix}\right\} = \frac{1}{4\pi\gamma(\ell-1)} \iint_{\sigma} \Delta g \left\{\begin{matrix}\cos m\lambda\\ \sin m\lambda\end{matrix}\right\} \bar{P}_{\ell m}(\sin\bar{\varphi})\, d\sigma \tag{1}$$

where γ is an average value of gravity and $\bar{P}_{\ell m}$ are the fully normalized potential coefficients of degree ℓ and order m as a function of geocentric latitude $\bar{\varphi}$, $d\sigma$ is a differential area, on a unit sphere, in which Δg is given. Because Δg are not given in differentially small blocks the integration in (1) can be approximated in several ways. We first write (1) in the following form.

$$\left\{\begin{matrix}\bar{C}_{\ell m}\\ \bar{S}_{\ell m}\end{matrix}\right\} = \frac{1}{4\pi\gamma(\ell-1)} \sum_{i=1}^{n} \overline{\Delta g}_i \left[\bar{P}_{\ell m}(\sin\bar{\varphi}_i)\left\{\begin{matrix}\cos m\lambda_i\\ \sin m\lambda_i\end{matrix}\right\}\right] \Delta\sigma_i \tag{2}$$

Here $\overline{\Delta g}_i$ is the mean anomaly in the area $\Delta\sigma_i$ and $\bar{P}_{\ell m}$ and (cos/sin) $m\lambda$ are evaluated at the mid-point $(\bar{\varphi}, \lambda)$ of the block. As $\Delta\sigma_i$ approaches $d\sigma$, equation (2) becomes equation (1).

An alternate procedure uses the following equation derived (Meissl, 1971) for a circular cap whose area is ΔS:

$$\bar{P}_{\ell m}(\sin\bar{\varphi}_i)\left\{\begin{matrix}\cos m\lambda_i\\ \sin m\lambda_i\end{matrix}\right\} = \frac{1}{\beta_\ell}\frac{1}{\Delta S_i}\iint_{S_i} \bar{P}_{\ell m}(\sin\bar{\varphi})\left\{\begin{matrix}\cos m\lambda\\ \sin m\lambda\end{matrix}\right\} dS \tag{3}$$

where β_ℓ is the Pellinen smoothing operator discussed extensively in Katsambalos (1979). β_ℓ is computed from:

$$\beta_\ell = \cot\left(\frac{\psi_0}{2}\right) \frac{P_{\ell 1}(\cos\psi_0)}{\ell(\ell+1)} \tag{4}$$

where ψ_0 is the radius of the cap. For rectangular blocks a corresponding β value can be computed by finding the radius of a cap having the same area as the block. We have for a block θ in size:

$$\sin\left(\frac{\psi_0}{2}\right) = \left(\frac{\theta\sin\theta}{4\pi}\right)^{\frac{1}{2}} \tag{5}$$

There is a difference between the gravitational spectra of a cap and a rectangular block but when (5) is used to achieve a correspondence the difference is small (Rapp, 1977).

If we use (3) in (2) we have:

$$\left\{\begin{matrix}\bar{C}_{\ell m}\\ \bar{S}_{\ell m}\end{matrix}\right\} = \frac{1}{4\pi\gamma(\ell-1)\beta_\ell} \sum_{i=1}^{n} \Delta g_i \iint_{\Delta S_i} \bar{P}_{\ell m}(\sin\varphi)\left\{\begin{matrix}\cos m\lambda\\ \sin m\lambda\end{matrix}\right\} dS \tag{6}$$

where ΔS_i is the rectangular block in which Δg_i is given. Either equation (2) or (5) are essentially equivalent, however, (6) is more stable at higher degrees (i. e. $> 180°/\theta°$) than (2) (Katsambalos, 1979).

In our first test of block size effects on potential coefficients we start from a 1°x 1° anomaly field used in previous studies by Rapp (1978). This set consisted of 50650 blocks with observed anomalies and 14150 values assumed to be zero. These anomalies were used in equation (2) to compute potential coefficients up to degree 36. The 1°x 1° anomalies were used to form a set of 1654 5° equal area anomalies by direct averaging of the 1°x 1° anomalies within the 5° block. These anomalies were then used in equation (6) although equation (2) could have been used with no substantial difference in the results.

We then compared the two coefficient sets by computing for each degree, the percentage difference, the undulation difference and the anomaly difference. These differences are shown in Table 1.

Table 1. Difference Between Potential Coefficients Computed from 5° and 1° Anomalies

n	%	ΔN (cm)	Δ(g) (mgals)	n	%	ΔN (cm)	Δ(g) (mgals)
				19	24	16	.45
2	1.4	37	.06	20	22	11	.33
3	1.3	29	.09	21	31	17	.53
4	3.0	30	.14	22	30	18	.57
5	2.0	17	.10	23	33	16	.55
6	3.2	19	.14	24	32	15	.52
7	4.2	20	.18	25	44	22	.83
8	5.7	18	.19	26	52	20	.79
9	5.6	14	.17	27	49	19	.76
10	8.7	20	.28	28	52	20	.84
11	7.4	13	.20	29	51	18	.76
12	13	14	.24	30	55	21	.95
13	11	17	.31	31	53	18	.84
14	17	16	.31	32	60	20	.95
15	15	14	.31	33	65	24	1.16
16	15	15	.36	34	62	24	1.24
17	15	13	.31	35	58	20	1.05
18	20	16	.41	36	75	24	1.31

Between degrees 2 and 36 the average percentage difference was 28%, the cumulative root mean square undulation difference was ±1.2 meters, and the cumulative root mean square anomaly difference was ±3.7 mgals.

We see from these results that there is substantial difference between the potential coefficients computed from the same fundamental data using two different block sizes at the higher degrees. For example, at degree 35 we see a 75% difference in the coefficients corresponding to an undulation difference of 24 cm and an anomaly difference of ±1.3 mgals. The overall difference between the two coefficient sets of ±1.2 m in undulation and ±3.7 mgals in anomaly is significant. It is clear that block size is an important factor in the accurate determination of potential coefficients.

Potential Coefficients from Combination Solutions

In most global gravity field determinations today satellite derived information is combined with terrestrial gravity data. Most of these combination solutions use 5° anomaly data. Based on our previous results it is now of interest to us to examine the effect of block size on combination solutions.

The type of combination solution to be used has been described in detail by Rapp (1978) where a combination of 1° x 1° anomalies and GEM 9 potential coefficients was carried out. The general model is written as follows:

$$F(L_{\ell a}, L_{xa}) = 0 \tag{7}$$

which is linearized to form the observation equation:

$$B_\ell V_\ell + B_x V_x + W = 0 \tag{8}$$

where

$$B_\ell = \frac{\partial F}{\partial L_\ell}, \quad B_x = \frac{\partial F}{\partial L_x}, \quad W = F(L_\ell, L_{xo}) \tag{9}$$

where L_ℓ are the actual observations (Δg's) and L_{xo} are the "observed" values of the quantities to be regarded as parameters (in our case potential coefficients) of the adjustment. If P_ℓ and P_x are the weight matrices for the observations and parameters respectively, we have for the correction to the observed parameters V_x:

$$V_x = -(B_x' M^{-1} B_x + P_x)^{-1} B_x' M^{-1} W \tag{10}$$

The corrections to the observed quantities (the gravity anomalies) V_ℓ are:

$$V_\ell = -P_\ell^{-1} B_\ell' \; M^{-1} (B_x V_x + W) \tag{11}$$

where

$$M = B_\ell P_\ell^{-1} B_\ell' \tag{12}$$

In our case

$$F = L_x{}^o - L_x{}^c \tag{13}$$

where $L_x{}^o$ are the given estimates of the potential coefficients obtained from satellite data and $L_x{}^c$ are the coefficients computed from (1) based on the global set of gravity anomalies.

Since $B_x = I$, (9) and (10) can be written:

$$V_x = -((B_\ell P_\ell^{-1} B_\ell')^{-1} + P_x)^{-1} (B_\ell P_\ell^{-1} B_\ell')^{-1} \; W \tag{14}$$

The elements of B_ℓ will depend on the form of (1) actually implemented. If we use equation (2) we have:

$$[B_\ell]_{PC} = \frac{-1}{4\pi\gamma(\ell-1)} \bar{P}_{\ell m}(\sin\bar{\varphi}_i) \begin{Bmatrix} \cos m\lambda_i \\ \sin m\lambda_i \end{Bmatrix} \Delta\sigma_i \tag{15}$$

If we use equation (6) for the potential coefficient determination we have

$$[B_\ell]_{PC} = \frac{-1}{4\pi\gamma(\ell-1)\beta_\ell} \begin{Bmatrix} \bar{A}_{\ell m} \\ \bar{B}_{\ell m} \end{Bmatrix} \tag{16}$$

where

$$\begin{Bmatrix} \bar{A}_{\ell m} \\ \bar{B}_{\ell m} \end{Bmatrix} = \iint_{\Delta S_i} \bar{P}_{\ell m}(\sin\bar{\varphi}) \begin{Bmatrix} \cos m\lambda \\ \sin m\lambda \end{Bmatrix} dS \tag{17}$$

Similar expressions hold for the anomaly coefficients $a_{0,0}$, $a_{1,0}$, $a_{1,1}$, $b_{1,1}$, which we wish to be held to near zero in the adjustment.

A combination solution was made with the 5° anomalies computed directly from the 1° x 1° anomalies described earlier. The potential coefficients used were the GEM 9 coefficients (Lerch et al., 1979) taken to degree 12 with P_ℓ assumed to be diagonal. A solution with the 1° x 1° anomalies has previously been described by Rapp (1978). In these solutions a set of adjusted anomalies was computed using equation (10). These anomalies were developed into potential coefficients for comparison purposes.

When dealing with 1° x 1° anomalies the coefficients of B_ℓ were computed from (15) while equation (16) was used with the 5° equal area anomalies. The adjusted coefficients were found from the adjusted anomalies using equation (2) for the 1° x 1° anomalies and equation (6) for the 5° equal area values.

The two adjusted coefficient sets were differenced to find, by degree the percentage difference, the undulation difference and the anomaly difference. These results are given in Table 2.

Table 2. Difference Between Potential Coefficients Computed from the Adjusted Anomalies of Combination Solutions Made with 5° and $1^\circ \times 1^\circ$ Anomalies and the GEM 9 Potential Coefficients.

n	%	ΔN (cm)	Δ(g) (mgals)	n	%	ΔN (cm)	Δ(g) (mgals)
				19	25	17	.46
2	.4	11	.02	20	23	12	.35
3	.7	14	.04	21	32	18	.55
4	.6	6	.03	22	30	18	.58
5	2.1	17	.11	23	32	16	.54
6	2.6	15	.11	24	32	15	.53
7	4.9	23	.21	25	45	23	.85
8	5.4	17	.18	26	53	21	.80
9	6.7	16	.20	27	49	19	.75
10	8.9	20	.28	28	51	20	.83
11	7.4	13	.19	29	53	18	.79
12	14	15	.25	30	56	22	.96
13	13	21	.38	31	54	18	.85
14	19	17	.35	32	59	20	.94
15	16	15	.33	33	64	23	1.16
16	17	17	.40	34	63	25	1.25
17	16	13	.32	35	60	21	1.08
18	21	16	.43	36	74	24	1.28

Between degrees 2 and 36 the average percentage difference was 29%, the cumulative root mean square undulation difference was ± 1.1 meters, and the cumulative root mean square anomaly difference was ± 3.8 mgals.

Comparison of Table 1 and Table 2 shows almost the same results after degree 4. Below that the changes in the combination solution are smaller by a factor of two from the previous solution. This is caused by the high a priori weighting of the GEM 9 coefficients at the lower degrees.

We see that if we wish to improve our geoid determination below the meter level and to avoid a significant perturbation of our potential coefficients we should not use 5° anomalies but rather use $1^\circ \times 1^\circ$ or 1° equal area anomalies. Such a procedure was used by Rapp (1978) but computer limitations (space and time) allowed only a strict adjustment to degree 12 to be carried out. If we are to use $1^\circ \times 1^\circ$ data a recasting of the combination programs to be used will be needed to allow a priori coefficients of all degrees (not just to 12) to be used in the combination solutions. Attempts have been made to improve the programs used in

Rapp (1978) but no substantial improvement in the rigorous adjustment programs have been found.

However, it is possible to carry out an adjustment using $1^\circ \times 1^\circ$ data using an approximate method that yields fast results and compromises the final solution in a small, and perhaps, acceptable way. Such a technique is described in the next section.

An Approximate Adjustment with $1^\circ \times 1^\circ$ Anomalies

Rapp (1969) has shown that under certain assumptions the combination procedure described in the previous section can be considerably simplified. This is done by assuming all equiangular anomalies have the same accuracy, m, and that the weights of the observations are assigned as follows:

$$[P_\ell] = \frac{\cos\varphi}{m^2} \tag{18}$$

where we assume P_ℓ is a diagonal matrix. If we assume that P_x is a diagonal matrix, an element of V_x (given in (14)) is:

$$[V_x] = \frac{-[W]}{1 + A\,[P_x]} \tag{19}$$

where W is given by (9) and A is:

$$A = \frac{m^2\, d\varphi\, d\lambda}{4\pi(\gamma(\ell-1))^2} \tag{20}$$

where $d\varphi$ and $d\lambda$ are the latitude and longitude increments of the block. The adjusted anomalies can be computed from (11) or the simpler expression (Rapp, 1969):

$$V_\ell = P_\ell^{-1}\, B_\ell'\, P_x\, V_x \tag{21}$$

The obvious advantage of this technique is that we are not required to form or invert the normal equations to obtain the adjusted coefficients or anomalies. However, to gain this advantage we assume all blocks have the same accuracy and weight according to (18).

We have carried out a combination solution with the approximate technique and compared the result with the rigorous procedure. This was done using the identical data set of Rapp (1978). Here the GEM 9 potential coefficients, taken to degree 12, were combined with the 50650 $1^\circ \times 1^\circ$ anomaly field assuming zero values for the blocks with no data. The adjusted anomalies were then developed into potential coefficients to degree 180. In carrying out the approximate solution we used a constant $1^\circ \times 1^\circ$ standard deviation (m) of ± 14.3 mgals which was the RMS value of the 64800 $1^\circ \times 1^\circ$ anomaly standard deviations used in the rigorous solution.

We examine the differences between the rigorous and approximate adjustment in several ways. Considering the adjusted coefficients to degree 12 only, we found a 5.9% difference, a 68 cm RMS undulation difference and a 0.7 mgal RMS difference.

When we compared the two 180 x 180 fields we found the average percentage difference to be 8.6%, the RMS undulation difference to be 80 cm and the RMS anomaly difference to be 2.0 mgals. The average percentage difference was a minimum of 0.6% at degree 3 and a maximum of 22% at degree 14. The average for degree 2 through 12 was: 6.0%; for 13 through 24: 15.4%; for 25 through 36: 11.2%; and for 37 through 48: 9.4%. Higher degree ranges had typical values of about 8%. From this latter comparison we conclude that the difference between the rigorous and approximate solutions, as judged on a global basis is small.

It is also informative to compare the individual $1^\circ \times 1^\circ$ adjusted anomalies from the two solutions. We found that the RMS anomaly difference was ± 2.2 mgals while the maximum difference was 90 mgals. The RMS residual was ± 3.9 mgals from the approximate solution and ± 4.2 mgals from the rigorous solution indicating no substantial difference in the residuals averaged globally.

The residuals themselves are highly dependent in the rigorous solution on the accuracy of the observed $1^\circ \times 1^\circ$ anomalies. In Table 3 we show the RMS residual as a function of the accuracy of the original anomalies for both the rigorous and approximate solution.

Table 3. RMS $1^\circ \times 1^\circ$ Residual Anomalies as a Function of the Original Anomaly Accuracies

Accuracy Range	Rigorous	Approximate	Number of Blocks
1 to 5mgals	±0.3 mgals	±1.8 mgals	5264
6 to 10	0.7	1.8	25945
11 to 15	2.1	2.7	8632
16 to 20	3.6	4.1	6394
21 to 25	5.1	5.2	4340
26 to 30	7.9	6.4	14190
31 to 35	27.9	2.4	35

We see the small residuals in the rigorous solution are at the small standard deviations while in the approximate solution the residuals are somewhat larger at the small standard deviations. Although all anomalies are given the same standard deviation in the approximate solution we can see some dependence in the residual magnitudes with respect to the standard deviations.

We also looked at the number of residuals by magnitudes as shown in Table 4.

Table 4. Number of $1^\circ \times 1^\circ$ Residuals Within A Specified Range

Range (mgals)	Rigorous	Approximate
0 to 2	47538	40291
2 to 4	7245	15096
4 to 6	3908	4210
6 to 8	2136	1625
8 to 10	921	813
10 to 12	598	586
12 to 14	564	446
14 to 16	301	497

This table shows that in most cases the number of residuals in each range is roughly the same in both solutions except for the 2 to 4 mgal range where the count differs by a factor of 2.

In comparing the adjusted anomalies from the two adjustments we found two anomalies where the residuals differed by 90 and 76 mgals. In both cases the assigned standard deviations, in the rigorous solution were ± 81 and ± 78 mgals. These extremely large standard deviations reflected the very large uncertainty in the original $1^\circ \times 1^\circ$ anomaly data. In the rigorous adjustment the residuals for these blocks were 93 and 79 mgals while in the approximate solution the residuals were both 3 mgals.

Timing Estimates

Computer time estimates for the rigorous and approximate combination solutions using $1^\circ \times 1^\circ$ anomalies have been made by Kostas Katsambalos based on our runs on the Amdahl 470 machine available at The Ohio State University. These times were estimated for carrying out an adjustment with a priori potential coefficients given to degree 20 and to degree 36. After the adjustment is made the adjusted anomalies can be developed into potential coefficients in two minutes which is thus a minor effort.

Table 5. Computer Time Estimates for Combination Solutions

n	Rigorous	Approximate
20	5 hours	4 minutes
36	47 hours	13 minutes

The time estimates clearly reflect the increased computational efforts for the rigorous solutions. Since the increase in computer time of the rigorous solution relative to the approximate solution is substantial, one must decide from the cost standpoint if the improved procedure is significantly more accurate than the approximate solution. Our previous discussions indicate that this will not be the case.

Conclusions

We have demonstrated that the use of 5° anomalies in potential coefficient determinations can introduce significant errors in these coefficients. Based on comparisons with solutions made with $1^\circ \times 1^\circ$ anomalies the percentage error was small at the lower degrees but increased to 74% at degree 36. In a combination solution with the GEM 9 coefficients the average percentage difference was 29% with an RMS undulation difference of ± 1.1 meters and a RMS anomaly difference of ± 3.8 mgals considering degrees 2 through 36. These substantial differences are caused by the distortion of the coefficients found from 5° blocks caused by the large size of the block.

These results strongly suggest that future combination solutions to degree 36 (or so) should be carried out with $1^\circ \times 1^\circ$ anomalies and not 5° data. However, the computer time to do this could be quite large so that an approximate technique for a combination solution was tested with the results compared to the rigorous solution. For degrees 2 through 36, the average percentage difference of the coefficients (rigorous vs approximate) was 14%, the RMS undulation difference was 79 cm and the RMS anomaly difference was ± 1.4 mgals. We see that the percent difference is about one-half that found in comparing the 5° solution with the 1° solution.

These results suggest that for highest accuracy in our combination solutions we should not use 5° anomalies but $1^\circ \times 1^\circ$ anomalies. Since a rigorous adjustment to degree 20 or 36 may be prohibitive in computer time an approximate combination solution can be made with the $1^\circ \times 1^\circ$ data that is very fast although some small differences over the optimum result will be found.

Finally, we should note that these combination solution discussions apply to one type of combination solution. Another form, most often used in practice (Rapp, 1969, Lerch, 1979) should be investigated to see if the same conclusions are valid.

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